# Chapter 11 – Revision of chapters 9-10

## Solutions to Review: Short-answer questions

**1 a** Sum of numbers showing is 5 means that one of the following four outcomes is observed: $\{(1, 4), (2, 3), (3, 2), (4, 1)\}$. Since there are 36 possible outcomes $n(\varepsilon) = 36$, and

$$\text{Pr(sum is 5)} = \frac{4}{36} = \frac{1}{9}$$

**b** $$\begin{aligned}\text{Pr(sum is not 5)} &= 1 - \text{Pr(sum is 5)}\\ &= 1 - \frac{1}{9}\\ &= \frac{8}{9}\end{aligned}$$

**2 a** Sample space: $\{348, 384, 438, 483, 843, 834\}$, $n(\varepsilon) = 6$

**b** Number is less than $500 = \{348, 384, 438, 483\}$, $n(\text{less than } 500) = 4$,
$\text{Pr(less than 500)} = \frac{4}{6} = \frac{2}{3}$

**c** $\text{Even} = \{348, 384, 438, 834\}$,
$n(\text{Even}) = 4$, $\text{Pr(Even)} = \frac{2}{3}$

**3 a** $$\begin{aligned}\text{Pr(Not red)} &= \text{Pr(Black)}\\ &= \frac{26}{52}\\ &= \frac{1}{2}\end{aligned}$$

**b** $$\begin{aligned}\text{Pr(Not an ace)} &= 1 - \text{Pr(Ace)}\\ &= 1 - \frac{1}{13}\\ &= \frac{12}{13}\end{aligned}$$

**4 a** Area circle $= \pi r^2$,

Area $A = \frac{\pi r^2}{4}$,

$\Pr(A) = \frac{\pi r^2}{4} \div (\pi r^2) = \frac{1}{4}$

**b** Area circle$= \pi r^2$,
Area $A = \frac{135}{360} \times \pi r^2 = \frac{3}{8}\pi r^2$
$\Pr(A) = \frac{3}{8} \times \pi r^2 \div \pi r^2 = \frac{3}{8}$

**5 a** Let,
$\Pr(1) = \Pr(2) = \Pr(3) = \Pr(5) = x$.
Then $\Pr(4) = 4x$, and $\Pr(6) = \frac{x}{2}$.
Since the sum of probabilities is 1,
$x + x + x + x + 4x + \frac{x}{2} = 1$.

So $x = \frac{2}{17}$.
Thus
$\Pr(1) = \Pr(2) = \Pr(3) = \Pr(5) = \frac{2}{17}$,
$\Pr(4) = \frac{8}{17}$, $\Pr(6) = \frac{1}{17}$

**b** $$\begin{aligned}\text{Pr( Not a 4)} &= 1 - \frac{8}{17}\\ &= \frac{9}{17}\end{aligned}$$

**6** Pr(hitting the blue circle) =
$\pi(10)^2 \div \pi(20)^2 = 100\pi \div 400\pi = \frac{1}{4}$

**7** $\Pr(B) = 0.3, \Pr(H) = 0.4$, and $\Pr(B \cap H) = 0.1$.

**a**

$$\begin{aligned}\Pr(B \cup H) &= \Pr(B) + \Pr(H) - \Pr(B \cap H)\\ &= 0.3 + 0.40.1\\ &= 0.6\end{aligned}$$

**b** $$\begin{aligned}\Pr(H|B) &= \frac{\Pr(H \cap B)}{\Pr(B)}\\ &= \frac{0.1}{0.3}\\ &= \frac{1}{3}\end{aligned}$$

**8**

| | Music | Not Music | |
|---|---|---|---|
| Painting | $\frac{15}{60}$ | $\frac{30}{60}$ | $\frac{45}{60}$ |
| Not Painting | $\frac{15}{60}$ | 0 | $\frac{15}{60}$ |
| | $\frac{30}{60}$ | $\frac{30}{60}$ | 1 |

From the table

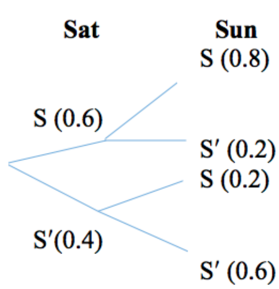
**a** $\frac{30}{60} = \frac{1}{2}$

**b** $\frac{45}{60} = \frac{3}{4}$

**c** $\frac{30}{60} = \frac{1}{2}$

**d** $\frac{15}{60} = \frac{1}{4}$

**9** Let $S$ be the event that the day is sunny.




**a** $$\begin{aligned}\Pr(\text{sunny all weekend}) &= \Pr(\text{SS})\\ &= 0.6 \times 0.8\\ &= 0.48\end{aligned}$$

**b**

$$\begin{aligned}\Pr(\text{Sunny on Sunday}) &= \Pr(SS \text{ or } S'S)\\ &= 0.6 \times 0.8 + 0.4 \times 0.2\\ &= 0.48 + 0.08\\ &= 0.56\end{aligned}$$

**10 a** $$\begin{aligned}\Pr(A \cap B) &= Pr(B|A)Pr(A)\\ &= 0.1 \times 0.5\\ &= 0.05\end{aligned}$$

**b** $$\begin{aligned}Pr(A|B) &= \frac{\Pr(A \cap B)}{Pr(B)}\\ &= \frac{0.05}{0.2}\\ &= 0.25\end{aligned}$$

**11** $A$ and $B$ are independent events, and $\Pr(A) = 0.4, \Pr(B) = 0.5$.

**a** $\Pr(A|B) = \Pr(A) = 0.4$ (since $A$ and $B$ are independent)

**b** $\Pr(A \cap B) = \Pr(A)\Pr(B)$

(since $A$ and $B$ are independent)

$$\begin{aligned}&= 0.4 \times 0.5\\ &= 0.2\end{aligned}$$

**c**

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B)\\ &= 0.4 + 0.5 - 0.2\\ &= 0.7\end{aligned}$$

**12** Since order is important there are $10 \times 9 \times 8 = 720$ ways

**13** Since order is not important, there are $\binom{52}{7} = 133784560$ different hands

**14** There are $\binom{12}{3} = 220$ different committees (without restrictions)

**a** If there is one girl then there are two boys. We can choose one girl from 7 girls and two boys from 5 boys in $\binom{7}{1} \times \binom{5}{2} = 7 \times 10 = 70$ ways.
Thus, $\Pr(\text{one girl}) = \dfrac{70}{220} = \dfrac{7}{22}$

**b** If there are two girls then there is one boy. We can choose two girls from 7 girls and one boy from 5 boys in $\binom{7}{2} \times \binom{5}{1} = 21 \times 5 = 105$ ways.
Thus $\Pr(\text{one girl}) = \dfrac{105}{220} = \dfrac{21}{44}$

## Solutions to Review: Multiple-choice questions

**1 E** Pr (success) $= \frac{1}{12}$ for each

Pr (both) $= \left(\frac{1}{12}\right)^2 = \frac{1}{144}$

**2 C** $\Pr(WB) + Pr(BW) = \left(\frac{2}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{13}{25}$

**3 E** Two dice, $\Pr(X > 12) = 0$,

$\Pr(X = 12) = \frac{1}{36}$

**4 B** $\Pr(G, B) + \Pr(B, G) = \frac{3}{7}\left(\frac{4}{6}\right) + \frac{4}{7}\left(\frac{3}{6}\right)$

$= \frac{4}{7}$

**5 E** $\Pr(X \cup Y) = \Pr(X) + \Pr(Y) - \Pr(X \cap Y)$

$= \Pr(Y') + \Pr(Y) - 0$

$= 1$

**6 C** Binomial, $n = 6, p = \frac{1}{6}$:

$\Pr(X \geq 1) = 1 - \Pr(X = 0)$

$= 1 - \left(\frac{5}{6}\right)^6$

**7 C** $\Pr(♥ \cup \mathrm{J}) = \Pr(♥) + \Pr(\mathrm{J}) - \Pr(\mathrm{J}♥)$

$= \frac{1}{4} + \frac{1}{13} - \frac{1}{52}$

$= \frac{16}{52} = \frac{4}{13}$

**8 B** $\Pr(R, R) = \left(\frac{k}{k+1}\right)\left(\frac{k-1}{k}\right)$

$= \frac{k-1}{k+1}$

**9 D** Bill: $n = 2, p = \frac{1}{2}$

Charles: $n = 4, p = \frac{1}{4}$

$\Pr(\geq 1) = 1 - \Pr(\text{none})$

Bill: $1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} = \frac{192}{256}$

Charles: $1 - \left(\frac{3}{4}\right)^4 = \frac{175}{256}$

Bill:Charles = 192:175

**10 D** Replace: $\Pr(A, A) = \left(\frac{4}{52}\right)^2 = \frac{1}{169}$

No replace: $\Pr(A, A) = \frac{4}{52}\left(\frac{3}{51}\right)$

$= \frac{1}{221}$

Ratio=221:169 = 17:13

**11 D** $N$(RAPIDS, vowels together)

= 2!(vowels) × 5!(cons + vowel group)

= 240

**12 E** $n$ from $(m + n)$: ${}^{m+n}\mathrm{C}_n = \frac{(m+n)!}{n!m!}$

**13 A** Choose 7 from 12 = ${}^{12}\mathrm{C}_7 = 792$

**14 E** 4 letters, 4 choices, replacement $= 4^4 = 256$

**15 E** $\Pr(O, O, O) = \frac{3}{6}\left(\frac{2}{5}\right)\frac{1}{4} = \frac{1}{20}$

**16 B** Person 1 has 6 × 10 possibilities. Person 2 enters by the same gate and can choose 9 exits.

**17 C** $\Pr(A \cap B) = \frac{1}{5}, \Pr(B) = \frac{1}{2},$

$\Pr(B|A) = \frac{1}{3}$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{1}{5} \div \frac{1}{2} = \frac{2}{5}$

$\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}$

$\Pr(A) = \frac{\Pr(A \cap B)}{\Pr(B|A)}$

$= \frac{1}{5} \div \frac{1}{3} = \frac{3}{5}$

**18 C** $\Pr(4, 6) + \Pr(6, 4) + \Pr(5, 5) = \frac{3}{36}$

**19 A** $\Pr(A, D, E, H, S) = \frac{1}{5!} = \frac{1}{120}$

**20 E** $\Pr(G,\ G) = \frac{4}{16}\left(\frac{3}{15}\right) = \frac{1}{20}$

**21 D** Binomial, $n = n,\ p = 0.15$

$\Pr(X \geq 1) = 1 - \Pr(X = 0)$

$\therefore 0.85^n < 0.1$

$\left(\frac{20}{17}\right)^n > 10, \therefore n > 14.2$

15 shots needed

**22 E** $(2x + 3)^5 = \sum_{i}^{5} \binom{5}{i} (2x)^{5-i}(3)^i$

Coefficient of $x^3$: let $i = 2$ in

$\binom{5}{i} (2)^{5-i}(3)^i$

$\binom{5}{2} (2)^{5-2}(3)^2 = \binom{5}{2} \times 2^3 \times 3^2$

## Solutions to Review: Extended-response questions

**1**

| Interval | No. of plants | Proportion | No. of plants > 30 cm | Proportion |
|---|---|---|---|---|
| (0, 10] | 1 | $\frac{1}{56}$ | | |
| (10, 20] | 2 | $\frac{2}{56}$ | | |
| (20, 30] | 4 | $\frac{4}{56}$ | | |
| (30, 40] | 6 | $\frac{6}{56}$ | 6 | $\frac{6}{49}$ |
| (40, 50] | 13 | $\frac{13}{56}$ | 13 | $\frac{13}{49}$ |
| (50, 60] | 22 | $\frac{22}{56}$ | 22 | $\frac{22}{49}$ |
| (60, 70] | 8 | $\frac{8}{56}$ | 8 | $\frac{8}{49}$ |
| Total | 56 | 1 | 49 | 1 |

Let $X$ be the height of the plants (in cm).

**a** **i** $\Pr(X > 50) = \frac{22}{56} + \frac{8}{56}$

$= \frac{30}{56} = \frac{15}{28} \approx 0.5357$

**ii** $\Pr(X > 50) + \Pr(X \leq 30) = \frac{30}{56} + \frac{1}{56} + \frac{2}{56} + \frac{4}{56}$

$= \frac{37}{56} \approx 0.6607$

**iii** $\Pr(X > 40|X > 30) = 1 - \Pr(X \leq 40|X > 30)$

$= 1 - \frac{6}{49} = \frac{43}{49} \approx 0.8776$

**b** $\Pr(F) = \frac{6}{7}$ and $\Pr(D) = \frac{1}{4}$

**i** $\Pr(F \cap D') = \Pr(F) \times \Pr(D')$

$= \frac{6}{7}(1 - \frac{1}{4}) = \frac{6}{7} \times \frac{3}{4}$

$= \frac{9}{14} \approx 0.6429$

**ii** $\Pr(F \cap D' \cap (X > 50)) = \Pr(F) \times \Pr(D') \times \Pr(X > 50)$

$$= \frac{9}{14} \times \frac{15}{28} = \frac{135}{392}$$

$$\approx 0.3444$$

**2 a** Possible choices

| $C$ | $B$ |
|---|---|
| 1 | any ball |
| 2 | 5 |
| 4 | 5 |
| 7 | no possible choice |

∴ probability that $B$ draws a higher number than $C$

$$= \frac{1}{3} + \frac{1}{6} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{1}{3} + \frac{1}{18} + \frac{1}{9}$$

$$= \frac{6 + 1 + 2}{18} = \frac{1}{2}$$

**b** Possible choices

| $B$ | $C$ | $A$ |
|---|---|---|
| 2 | 1 | 3 or 6 |
| 2 | 2 | 3 or 6 |
| 2 | 4 | 6 |
| 2 | 7 | no possible choice |
| 5 | 1 | 6 |
| 5 | 2 | 6 |
| 5 | 4 | 6 |
| 5 | 7 | no possible choice |

∴ probability that $A$ draws a higher number than $B$ or $C$

$$= \frac{2}{3} \times \frac{1}{3} \times \frac{5}{6} + \frac{2}{3} \times \frac{1}{6} \times \frac{5}{6} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{6} \times \frac{1}{6} + \frac{1}{3} \times \square \frac{1}{3} \times \frac{1}{6}$$

$$= \frac{10}{54} + \frac{10}{108} + \frac{2}{54} + \frac{1}{54} + \frac{1}{108} + \frac{1}{54}$$

$$= \frac{20 + 10 + 4 + 2 + 1 + 2}{108} = \frac{39}{108} = \frac{13}{36}$$

**3 a**

| 1st card | | 2nd card | Probability |
|---|---|---|---|
| $\frac{2}{8}$ A | — | $\frac{3}{7}$ L | $\frac{6}{56}$ |
| $\frac{1}{8}$ E | — | $\frac{3}{7}$ L | $\frac{3}{56}$ |
| $\frac{3}{8}$ L | — | $\frac{2}{7}$ L | $\frac{6}{56}$ |
| $\frac{1}{8}$ P | — | $\frac{3}{7}$ L | $\frac{3}{56}$ |
| $\frac{1}{8}$ R | — | $\frac{3}{7}$ L | $\frac{3}{56}$ |

$$\text{Probability that second card bears L} = \frac{6}{56} + \frac{3}{56} + \frac{6}{56} + \frac{3}{56} + \frac{3}{56} = \frac{21}{56} = \frac{3}{8}$$

**b** $\Pr(A, L, E) = \frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} = \frac{1}{56}$

**c**

| 1st card | 2nd card | | 3rd card | Probability |
|---|---|---|---|---|
| $\frac{2}{8}$ A | $\frac{3}{7}$ L | — | $\frac{1}{6}$ E | $\frac{6}{336}$ |
| | $\frac{1}{7}$ E | — | $\frac{3}{6}$ L | $\frac{6}{336}$ |
| $\frac{3}{8}$ L | $\frac{2}{7}$ A | — | $\frac{1}{6}$ E | $\frac{6}{336}$ |
| | $\frac{1}{7}$ E | — | $\frac{2}{6}$ A | $\frac{6}{336}$ |
| $\frac{1}{8}$ E | $\frac{2}{7}$ A | — | $\frac{3}{6}$ L | $\frac{6}{336}$ |
| | $\frac{3}{7}$ L | — | $\frac{2}{6}$ A | $\frac{6}{336}$ |

$$\Pr(A, L, E \text{ in any order}) = \frac{36}{336} = \frac{3}{28}\left(\text{or } 3! \times \frac{1}{56} = \frac{6}{56} = \frac{3}{28}\right)$$

**d**

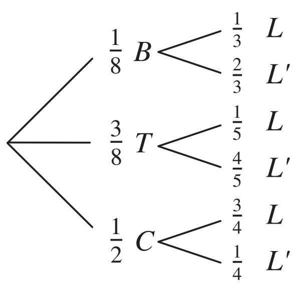
| 1st card | | 2nd card | Probability |
|---|---|---|---|
| $\frac{2}{8}$ A | — | $\frac{6}{7}$ not A | $\frac{12}{56}$ |
| $\frac{1}{8}$ E | — | $\frac{7}{7}$ not E | $\frac{7}{56}$ |
| $\frac{3}{8}$ L | — | $\frac{5}{7}$ not L | $\frac{15}{56}$ |
| $\frac{1}{8}$ P | — | $\frac{7}{7}$ not P | $\frac{7}{56}$ |
| $\frac{1}{8}$ R | — | $\frac{7}{7}$ not R | $\frac{7}{56}$ |

Probability that first two cards bear different letters

$$= \frac{12}{56} + \frac{7}{56} + \frac{15}{56} + \frac{7}{56} + \frac{7}{56}$$

$$= \frac{48}{56} = \frac{6}{7}$$

**4** **a** Let $L$ be the event ' an employee is late', $B$ the event 'travels by bus', $T$ the event 'travels by train', and $C$ the event 'travels by car'.




$$\Pr(L) = \Pr(L \cap B) + \Pr(L \cap T) + \Pr(L \cap C)$$

$$= \Pr(L|B) \times \Pr(B) + \Pr(L|T) \times \Pr(T) + \Pr(L|C) \times \Pr(C)$$

$$= \frac{1}{8} \times \frac{1}{3} + \frac{3}{8} \times \frac{1}{5} + \frac{1}{2} \times \frac{3}{4}$$

$$= \frac{1}{24} + \frac{3}{40} + \frac{3}{8}$$

$$= \frac{5 + 9 + 45}{120}$$

$$= \frac{59}{120} \approx 0.4917$$

**b** $\Pr(C|L) = \dfrac{\Pr(C \cap L)}{\Pr(L)} = \dfrac{\Pr(L|C) \times \Pr(C)}{\Pr(L)}$

$$= \frac{\frac{3}{8}}{\frac{59}{120}} = \frac{3 \times 120}{8 \times 59}$$

$$= \frac{45}{59} \approx 0.7627$$

**5 a i**

$$m + 10 = 40$$
$$\therefore \quad m = 30$$
$$q + 10 = 45$$
$$\therefore \quad q = 35$$
$$m + q + s + 10 = 100$$
$$\therefore \quad s = 100 - 10 - m - q$$
$$= 100 - 10 - 30 - 35$$
$$\therefore \quad s = 25$$

**ii** $m + q = 30 + 35$

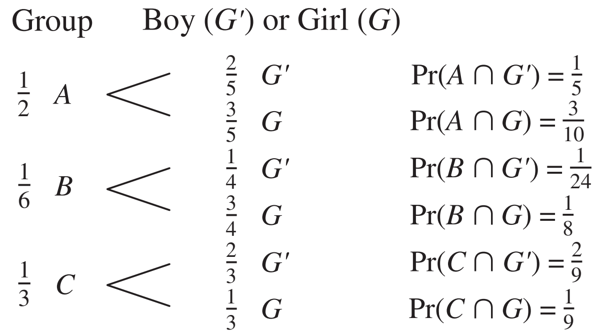
$= 65$

**b** Let $H$ be the event 'History is taken'
Let $G$ be the event 'Geography is taken'.

$$\Pr(H \cap G') = \frac{m}{100} = \frac{30}{100} = 0.3$$

**c** $$\Pr(G|H') = \frac{\Pr(G \cap H')}{\Pr(H')} = \frac{\frac{q}{100}}{\frac{100 - m - 10}{100}} = \frac{q}{90 - m} = \frac{35}{60} = \frac{7}{12} \approx 0.5833$$

**6** Let $A$ be the event 'Group $A$ is chosen', $B$ be the event 'Group $B$ is chosen' and $C$ be the event ' Group $C$ is chosen'




**a** $\Pr(G') = \Pr(G' \cap A) + \Pr(G' \cap B) + \Pr(G' \cap C)$

$$= \frac{1}{5} + \frac{1}{24} + \frac{2}{9}$$

$$= \frac{216 + 45 + 240}{1080}$$

$$= \frac{501}{1080} = \frac{167}{360} \approx 0.639$$

**b** **i** $\Pr(A|G) = \dfrac{\Pr(A \cap G)}{\Pr(G)}$

$$= \frac{\Pr(A \cap G)}{\Pr(A \cap G) + \Pr(B \cap G) + \Pr(C \cap G)}$$

$$= \frac{\frac{3}{10}}{\frac{3}{10} + \frac{1}{8} + \frac{1}{9}}$$

$$= \frac{\frac{3}{10}}{\frac{108 + 45 + 40}{360}}$$

$$= \frac{3}{10} \times \frac{360}{193}$$

$$= \frac{108}{193} \approx 0.596$$

Note: $\Pr(G)$ can also be found by *calculating* $1 - \Pr(G')$ or directly from the tree diagram.

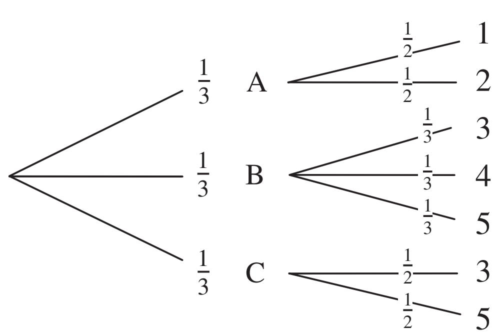
**ii** $\Pr(B|G) = \dfrac{\Pr(B \cap G)}{\Pr(G)}$

$$= \frac{\frac{1}{8}}{\frac{193}{360}}$$

$$= \frac{1}{8} \times \frac{360}{193}$$

$$= \frac{45}{193} \approx 0.332$$

**7 a**




**i** Pr(4 drawn) = Pr(B chosen and 4 drawn)

$$= \frac{1}{9}$$

$$\approx 0.1111$$

**ii** Pr(3 drawn) = Pr(B chosen and 3 drawn) + Pr(C chosen and 3 drawn)

$$= \frac{1}{9} + \frac{1}{6}$$

$$= \frac{5}{18}$$

$$\approx 0.2778$$

**b** **i** Pr(balls drawn by David and Sally are both 4)

= Pr(B chosen and 4 drawn) × Pr(B chosen and 4 drawn)

$$= \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$$

$$\approx 0.0123$$

**ii** Pr(David and Sally both draw balls numbered 3 from the same bag)

$= \Pr(\text{B chosen and 3 drawn}) \times \Pr(\text{B chosen and 3 drawn})$

$\quad + \Pr(\text{C chosen and 3 drawn}) \times \Pr(\text{C chosen and 3 drawn})$

$$= \frac{1}{9} \times \frac{1}{9} + \frac{1}{6} \times \frac{1}{6}$$

$$= \frac{1}{81} + \frac{1}{36}$$

$$= \frac{36 + 81}{2916}$$

$$= \frac{117}{2916} = \frac{13}{324}$$

$$\approx 0.0401$$

**8 a** Pr(total score = 23) = $\Pr(A = 8) \times \Pr(B = 6) \times \Pr(C = 9)$ as there are no other ways of achieving 23, and spins are independent.

$$\therefore \quad \Pr(\text{total score} = 23) = \frac{4}{10} \times \frac{7}{10} \times \frac{3}{10}$$

$$= \frac{84}{1000}$$

$$= 0.084$$

**b** Possible combinations for Player $B$ to score more than Player $C$

| Player $B$ | Player $C$ |
|---|---|
| 3 | 2 |
| 6 | 2 |
| 6 | 5 |

$$\Pr(B > C) = \Pr(B = 3, C = 2) + \Pr(B = 6, C = 2) + \Pr(B = 6, C = 5)$$

$$= \Pr(B = 3) \times \Pr(C = 2) + \Pr(B = 6) \times \Pr(C = 2) + \Pr(B = 6) \times \Pr(C = 5)$$

$$= \frac{3}{10} \times \frac{1}{10} + \frac{7}{10} \times \frac{1}{10} + \frac{7}{10} \times \frac{6}{10}$$

$$= \frac{3+7+42}{100}$$
$$= \frac{52}{100}$$
$$= 0.52$$

**c** Possible combinations for Player $C$ to score more than Player $A$

| Player $C$ | Player $A$ |
|---|---|
| 2 | 1 |
| 5 | 1 |
| 9 | 1 |
| 5 | 4 |
| 9 | 4 |
| 9 | 8 |

$$\begin{aligned}\Pr(C > A) &= \Pr(C = 2, A = 1) + \Pr(C = 5, A = 1) + \Pr(C = 9, A = 1)\\ &\quad + \Pr(C = 5, A = 4) + \Pr(C = 9, A = 4) + \Pr(C = 9, A = 8)\\ &= \Pr(C = 2) \times \Pr(A = 1) + \Pr(C = 5) \times \Pr(A = 1) + \Pr(C = 9)\\ &\quad \times \Pr(A = 1) + \Pr(C = 5) \times \Pr(A = 4) + \Pr(C = 9) \times \Pr(A = 4)\\ &\quad + \Pr(C = 9) \times \Pr(A = 8)\\ &= \frac{1}{10} \times \frac{2}{10} + \frac{6}{10} \times \frac{2}{10} + \frac{3}{10} \times \frac{2}{10} + \frac{6}{10} \times \frac{4}{10} + \frac{3}{10} \times \frac{4}{10} + \frac{3}{10} \times \frac{4}{10}\\ &= \frac{2+12+6+24+12+12}{100}\\ &= \frac{68}{100}\\ &= 0.68\end{aligned}$$

**9**


**a** There are $3 \times 4 \times 5 = 60$ different routes from A to $D$.

**b** There are $2 \times 2 \times 2 = 8$ routes without roadworks.

**c** $\Pr(\text{roadworks at each stage}) = \frac{1}{3} \times \frac{2}{4} \times \frac{3}{5}$

$$= \frac{1}{10} = 0.1$$

**10** Let $A$ be the event '$A$ hits the target', $B$ be the event '$B$ hits the target', and $C$ be the event '$C$ hits the target'.

$\therefore \Pr(A) = \frac{1}{5}, \Pr(B) = \frac{1}{4}, \Pr(C) = \frac{1}{3}$

**a** $\Pr(A \cap B \cap C) = \Pr(A) \times \Pr(B) \times \Pr(C)$ as $A$, $B$, $C$ are independent

$$= \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3}$$

$$= \frac{1}{60} \approx 0.0167$$

**b** $\Pr(A') = \frac{4}{5}, \Pr(B') = \frac{3}{4}$

$$\Pr(A' \cap B' \cap C) = \Pr(A') \times \Pr(B') \times \Pr(C)$$

$$= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}$$

$$= \frac{1}{5} = 0.2$$

**c** Pr(at least one shot hits the target) = 1 − Pr (no shot hits the target)

$$= 1 - \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$$

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5} = 0.6$$

**d** $\Pr(C|\text{only one shot hits the target})$

$$= \frac{\Pr(C \cap A' \cap B')}{\Pr(A \cap B' \cap C') + \Pr(A' \cap B \cap C') + \Pr(A' \cap B' \cap C)}$$

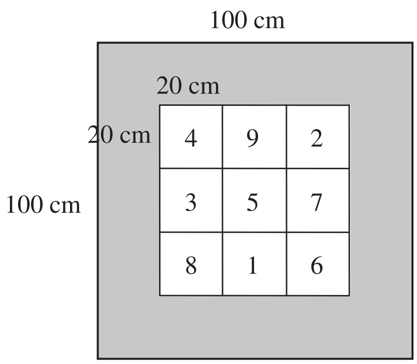
$$= \frac{\frac{1}{3} \times \frac{4}{5} \times \frac{3}{4}}{\frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}}$$

$$= \frac{\frac{12}{60}}{\frac{6}{60} + \frac{8}{60} + \frac{12}{60}}$$

$$= \frac{12}{26}$$

$$= \frac{6}{13} \approx 0.4615$$

**11** **a**




**i** Area of large outer square = $100 \times 100 = 10\,000$ cm$^2$.

**ii** Area of one inner square = $20 \times 20 = 400$ cm$^2$.

**iii** Area of shaded region = $10\,000 - 9 \times 400 = 6400$ cm$^2$.

**b** **i** Pr(one dart will score 7) $= \dfrac{400}{10\,000}$

$= 0.04$

(i.e. area of small square marked 7 divided by area of large square)

**ii** Pr(at least 7) = Pr(7) + Pr(8) + Pr(9)

$= 3 \times 0.4 = 0.12$

**iii** Pr(score will be 0) $= \dfrac{\text{area of shaded region}}{\text{total area of board}}$

$= \dfrac{6400}{10\,000} = 0.64$

**c** **i** To get 18 from two darts, 9 and 9 need to be thrown.

Pr(18) $= 0.04 \times 0.04$

$= 0.0016$

**ii** Throws to score 24 are 6, 9, 9 or 7, 8, 9 or 8, 8, 8 in any order, i.e. possible throws

| | | | | | |
|---|---|---|---|---|---|
| 6 | 9 | 9 | 7 | 8 | 9 |
| 7 | 9 | 8 | 8 | 7 | 9 |
| 8 | 8 | 8 | 8 | 9 | 7 |
| 9 | 6 | 9 | 9 | 7 | 8 |
| 9 | 8 | 7 | 9 | 9 | 6 |

There are 10 winning combinations.

$$\begin{aligned}\text{Pr(a winning combination)} &= (0.04)^3\\ \therefore \text{Pr (scoring 24)} &= 10 \times (0.04)^3\\ &= 10 \times 0.000064\\ &= 0.00064\end{aligned}$$

**12 a** The possible choices are

| $c$ | $b$ |
|---|---|
| 8 | 11 |
| 3 | 11 |
| 3 | 7 |

$$\begin{aligned}\Pr(c < b) &= \Pr(c = 8, b = 11) + \Pr(c = 3, b = 11) + \Pr(c = 3, b = 7)\\ &= \frac{1}{3} \times \frac{1}{6} + \frac{2}{3} \times \frac{1}{6} + \frac{2}{3} \times \frac{1}{3}\\ &= \frac{1}{18} + \frac{2}{18} + \frac{2}{9}\\ &= \frac{7}{18} \approx 0.3889\end{aligned}$$

**b** Possible choices

| $b$ | $c$ | $a$ |
|---|---|---|
| 1 | 3 | 6 |
| 1 | 3 | 10 |
| 1 | 8 | 10 |
| 7 | 3 | 10 |
| 7 | 8 | 10 |

$$\begin{aligned}\Pr(a > \text{ both } b \text{ and } c) &= \Pr(a = 6, b = 1, c = 3) + \Pr(a = 10, b = 1, c = 3)\\ &\quad + \Pr(a = 10, b = 1, c = 8) + \Pr(a = 10, b = 7, c = 3)\\ &\quad + \Pr(a = 10, b = 7, c = 8)\\ &= \frac{2}{3} \times \frac{1}{2} \times \frac{2}{3} + \frac{1}{6} \times \frac{1}{2} \times \frac{2}{3} + \frac{1}{6} \times \frac{1}{2} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{3} \times \frac{2}{3}\\ &\quad + \frac{1}{6} \times \frac{1}{3} \times \frac{1}{3}\\ &= \frac{4}{18} + \frac{2}{36} + \frac{1}{36} + \frac{2}{54} + \frac{1}{54}\\ &= \frac{24 + 6 + 3 + 4 + 2}{108}\\ &= \frac{39}{108}\\ &= \frac{13}{36} \approx 0.3611\end{aligned}$$

**c** Possible choices

| $a$ | $b$ | $c$ |
|---|---|---|
| 0 | 1 | 3 |
| 0 | 1 | 8 |
| 0 | 7 | 8 |
| 6 | 1 | 8 |

$$\begin{aligned}\Pr(c > a + b) &= \Pr(c = 3,\ a = 0,\ b = 1) + \Pr(c = 8,\ a = 0,\ b = 1)\\ &\quad + \Pr(c = 8,\ a = 0,\ b = 7) + \Pr(c = 8,\ a = 6,\ b = 1)\\ &= \frac{2}{3} \times \frac{1}{6} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{6} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{6} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{2}\\ &= \frac{2}{36} + \frac{1}{36} + \frac{1}{54} + \frac{2}{18}\\ &= \frac{6 + 3 + 2 + 12}{108}\\ &= \frac{23}{108} \approx 0.2130\end{aligned}$$